Theorem

Fig

$$\overline{az + bz^2 + cz^3 + dz^4 + ez^5 + fz^6 + gz^7 -}$$

$$= a^m z^m + \frac{m}{1} a^{m-1} b z^{m+1} \quad + \frac{m}{1} \times \frac{m-1}{2} a^{m-2} b^2 z^{m+2} \quad + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} a^{m-3}$$

$$+ \frac{m}{1} a^{m-1} c$$

$$+ \frac{m}{1} \times \frac{m-1}{1} a^{m-2}$$

$$+ \frac{m}{1} a^{m-1}$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} a^{m-5} b^5 z^{m+5}$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{1} a^{m-4} b^3 c$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{1} a^{m-3} b^2 d$$

$$+ \frac{m}{1} \times \frac{m-1}{1} \times \frac{m-2}{2} a^{m-3} b c^2$$

$$+ \frac{m}{1} \times \frac{m-1}{1} a^{m-2} b e$$

$$+ \frac{m}{1} \times \frac{m-1}{1} a^{m-2} c d$$

$$+ \frac{m}{1} a^{m-1} f$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} \times$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{1} \times$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times$$

$$+ \frac{m}{1} \times \frac{m-1}{1} \times$$

$$+ \frac{m}{1} \times$$

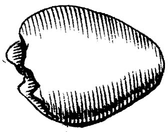
$$+ \frac{m}{1} \times \frac{m-1}{2} \times$$

$$+ \frac{m}{1} \times$$

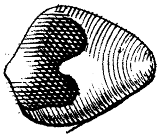
$$+ \frac{m}{1} \times$$

$$+$$

4



3



2

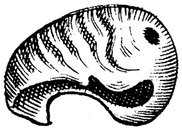
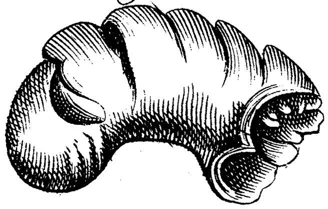


Fig: 1



The Royal Society is collaborating with JSTOR to digitize, preserve, and extend access to
Philosophical Transactions (1683-1775).
www.jstor.org

Fig. 5. Transact: N° 230.

$$\overline{gz^{7}+hz^{8}+iz^{9}\ \&c}^{\,m} =$$

$a^{m-3}b^{3}z^{m+3} \quad + \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}a^{m-4}b^{4}z^{m+4}$

${}^{m-2}bc \quad + \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{1}a^{m-3}b^{2}c$

${}^{m-1}d \quad + \frac{m}{1}\times\frac{m-1}{1}a^{m-2}bd$

$+ \frac{m}{1}\times\frac{m-1}{2}a^{m-2}c^{2}$

$+ \frac{m}{1}a^{m-1}e$

$\frac{-4}{5}\times\frac{m-5}{6}a^{m-6}b^{6}z^{m+6}\ \&c$

$\frac{-3}{4}\times\frac{m-4}{1}a^{m-5}b^{4}c$

$\frac{-2}{3}\times\frac{m-3}{1}a^{m-4}b^{3}d$

$\frac{-2}{1}\times\frac{m-3}{2}a^{m-4}b^{2}c^{2}$

$\frac{-1}{2}\times\frac{m-2}{1}a^{m-3}b^{2}e$

$\frac{-1}{1}\times\frac{m-2}{1}a^{m-3}bcd$

$\frac{m}{1}\times\frac{m-1}{1}a^{m-2}bf$

$\frac{-1}{2}\times\frac{m-2}{3}a^{m-3}c^{3}$

$\frac{m}{1}\times\frac{m-1}{1}a^{m-2}ce$

$\frac{m}{1}\times\frac{m-1}{2}a^{m-2}d^{2}$

$+\frac{m}{1}a^{m-1}g$

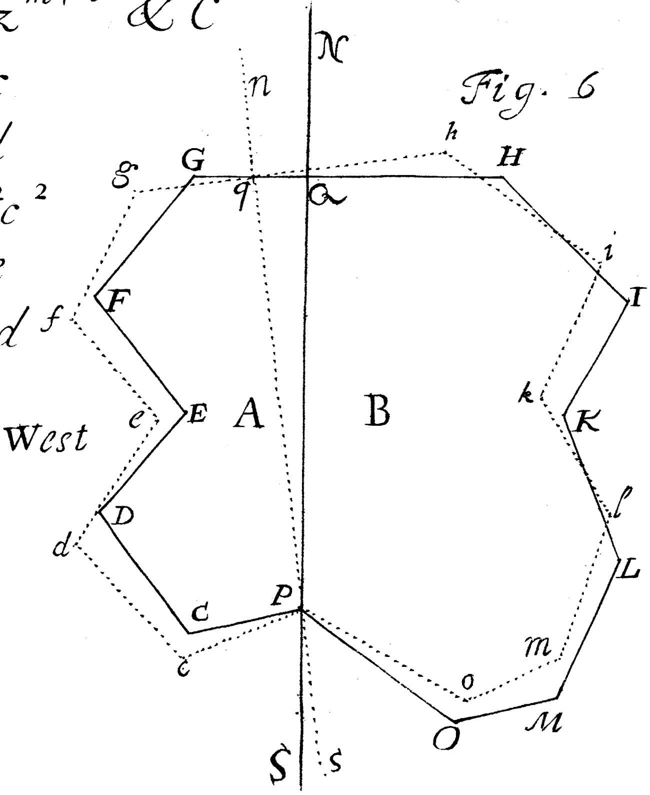


Fig. 6

that whatever is ſaid of one may be ſaid of t'other. However I think to give ſometime a more formal Demonſtration of it.

See the Theorem *Fig.* 5.

IV. *A Demonſtration of an Error committed by common Surveyors in comparing of Surveys taken at long Intervals of Time ariſing from the Variation of the Magnetick Needle*, *by* William Molyneux *Eſq*; *F. R. S.*

THE Variation of the Magnetick Needle is ſo commonly known, that I need not inſiſt much on the Explication thereof, 'tis certain that the true Solar Meridian, and the Meridian ſhewn by a Needle, agree but in very few places of the World; and this too, but for a little time (if a Moment) together. The Difference between the true Meridian and Magnetick Meridian perpetually varying and changing in all Places and at all Times; ſometimes to the Eaſtward and ſometimes to the Weſtward.

On which account 'tis impoſſible to compare two Surveys of the ſame place, taken at diſtant times, by Magnetick Inſtruments (ſuch as the *Circumferentor*, by which the *Down Survey*, or Sir *William Petty*'s Survey of *Ireland* was taken) without due allowance be made for this Variation. To which purpoſe we ought to know the Difference between the Magnetick Meridian and true Meridian at that time of the Down Survey, and the ſaid Difference at the time, when we make a New Survey to compare with the Down Survey.

But here I would not be underſtood, as if I propoſed hereby to ſhew, that a Map of the ſame place, taken by Magnetick Inſtruments at never ſo diſtant times, ſhould not at one time give the ſame *Figure* and *Contents* as at another time. This certainly it will do moſt exactly, the variation of the Needle having nothing to do either in the *Shape* or *Contents* of the Survey. All that is affected thereby, is, the Bearings of the Lines run by the Chain, and the Boundaries between Neighbours. And how this may cauſe a conſiderable Error (unleſs due allowance be made for it) is what I ſhall prove moſt fully.

In order to which, let us ſuppoſe that about the Year 1657. (at which time the Down Survey was taken) the Magnetick Meridian and true Meridian did agree at *Dublin*, or pretty nigh all over *Ireland*; that is to ſay, that there was no Variation. And indeed by Experiment it was at that time found, as I am well aſſur'd, that at *Dublin* it was hardly half a Degree.

Let us ſuppoſe that in the Year 1695. the Variation was 7 Degrees from the North to the Weſtward: that it was really ſo, I believe I am pretty well aſſured, from an Experiment thereof made by my ſelf with all diligence. But this is not material, let us now only ſuppoſe it.

Let *A B* repreſent the Survey of two Town Lands, one in the poſſeſſion of *A*, and t'other in the poſſeſſion of *B*, which we will call *A* Town-Land and *B* Town-Land, taken by the Down Survey, *Anno* 1657. when there was no Variation.

Let the Line *N S* running through the Point *P* be the true Meridian, and conſequently the Magnetick Meridian alſo at that time, becauſe of the ſuppoſed no Variation, and let this Line *N S* be alſo the Boundary between the two Town-Lands *A* and *B*.

In

In the Year 1695. when the Variation is 7 Degrees from the North to the Weſtward, *B* having a Map of the Down Survey, and being ſuſpicious that his Neighbour *A* had incroached on him by a Ditch *P Q*, imploys a Surveyor to inquire into the Matter: The Surveyor finds by his Map that the Boundary between *B* and his Neighbour *A* run from the Point *P* through a Meadow directly according to the Magnetick Meridian *S P N*; but obſerving the Ditch *P Q* caſt up much to the Eaſtward of the preſent Magnetick Meridian, he concludes that *A*. has incroached on *B*, and that the Ditch ought to have been caſt up alongſt the Line *P q*, the Angle *Q P q* being an Angle of 7 Degrees, that is the preſent Variation of the Needle; and the Line *P q* the preſent Magnetick Meridian: For which Variation, not making any allowance, he poſitively determines that *B*. has all the Land in the Triangle *Q P q*, more than he ought to have; and that his Ditch ought to run alongſt the Line *P q*.

'Tis true indeed, if the Surveyor go the whole Surround of the Lands *A* and *B* he will find their Figure and Contents exactly agreeable to the Map here expreſſed. But then the Bearings of the Lines are all 7 Degrees different from the Bearings in the Map, and they will run in and out upon the adjacent Neighbouring Lands, and cauſe endleſs Differences between their Poſſeſſors; as is manifeſt from the Figure: wherein the prickt Lines repreſent the Diſagreement in the Bearings of the Lines, protracted from the Point *P*; and we ſee *A* incroaching on his Neighbours on the Weſtward, as he incroaches on *B*, and *B*'s Eaſtward Neighbours incroaching on him, and ſo forward and clear round. Whereas, by a due allowance for the Variation of the Needle, all this Confuſion and Diſagreement is avoided, and every thing hits right.

Thus,

Thus for inſtance in the Caſe before us, knowing that the Magnetick Variation has cauſed the preſent Magnetick Meridian to fall in the Line *n q P s*, 7 Degrees from the North to the Weſtward; to Reduce this to the Magnetick Meridian at the time of the Down Survey, I muſt make the Meridian of my Map to fall 7 Degrees to the Eaſtward of my Magnetick Meridian; as we ſee the Line *P Q* falls 7 Degrees to the Eaſtward of the Line *P q*.

What is here ſaid on ſuppoſition that the Magnet had no Variation at the time of the firſt Survey taken, and that it had 7 Degrees variation Weſtward at the time of the ſecond Survey, may eaſily be accommodated to the ſuppoſal of any other Variations at the firſt and ſecond Surveys, *Mutatis mutandis*, for knowing the Variations we know their Difference; and if we know their Difference, this gives us the Angle *Q P q*, by which we reduce them to each other.

The beſt way therefore to make Maps invariable, conſtant, and everlaſting, were for the Surveyors, who uſe Magnetick Inſtruments to make always allowance for the Magnetick Variation, and to protract and lay down their Plats by the true Meridian. This the wary Sailor is fully convinced of: and therefore in Steering his Courſe, he conſtantly allows for the preſent Variation, which he obſerves by the *Azimuth* Compaſs, or elſe he would miſs his appointed Harbour oftner then he would hit it: For no two Points on the Globe keep the ſame Bearing to each other by the Magnetick Meridian for any time together. And though the Variation be ſlow, yet in a long Courſe, or in times pretty diſtant, it may cauſe vaſt Errors, unleſs allowed for. Thus for inſtance, Suppoſe in the Year 1660. a Sailor had ſteered from the Lands-end of *England* to Cape *Feniſter* in *Spain*, by his Magnetick Compaſs a direct South Courſe; and

that

that at that time there were no Varation. Afterwards, *Anno* 1700. when there was (ſuppoſe) 7 Degrees of Variation from the North to the Weſtward, another Sailor intending to make the ſame Paſſage, ſteers directly the ſame Southerly Courſe by his Magnetick Compaſs; I ſay, this laſt Seaman will be carried far into the *Bay of Biſcay* to the Eaſtward, and will miſs of his deſired Port by many Leagues; but if in his Courſe he hath allowed for this Variation, and inſtead of ſailing a direct Southerly Courſe by his Compaſs, he had ſteer'd 7 Degrees from the South to the Weſtward, he had hit his Point. Whether theſe be the true Bearings of theſe two Places, it matters not: we go on the Suppoſition that they are.

Perhaps it may be objected, That Surveys may be taken without Magnetick Inſtruments, and that therefore this Error ariſing from the Magnetick Variation, and Change of the Bearing of Lines, may be avoided. To which I Anſwer, firſt, That granting a Survey may be taken *without* Magnetick Inſtruments, this is nothing againſt what we have laid down relating to Surveys that are taken *with* Magnetick Inſtruments, as the Down Survey actually was, and moſt Surveys at preſent actually are taken therewith. Secondly, Though a Survey may be taken truly without Magnetick Inſtruments, ſo as to ſhew the exact Angles and Lines of the Plat, and conſequently the true Contents, yet this will not give the true Bearings of the Lines, or ſhew my Poſition in relation to my Neighbours, or the other parts of the Country. This muſt be ſupply'd by the Magnet, or ſomething equivalent thereto, as finding a true Meridian Line on your Land by Celeſtial Obſervation. And I doubt not but the ancient *Ægyptians*, before the Diſcovery of the Magnet were forced to ſome ſuch Expedient in their Surveys and Applotments of

Lands, between Neighbour and Neighbour, after the Inundations of the *Nile*, which, we are told, gave the firſt Original to Geometry and Surveying. Abſolute Neceſſity and Uſe having introduced theſe, as Delight and Diverſion introduced Aſtronomy amongſt the *Chaldeans*.

And this brings me to another Objection which may be made againſt the Inſtance before laid down: It may be ſaid, That certainly the Surveyor which *B* imployed was very ignorant, who would chooſe to judge of the Line *P Q*. rather by its bearing than by determining the Point *Q*, by meaſuring from *H* and *G*. To this I anſwer, What if both the Points *H* and *G* were vaniſh'd ſince the Down Survey was taken? What if the whole face of the Country were chang'd, ſave only the Point *P* and the Line *P Q*? How ſhall the Surveyor then judge of the Line *P Q* but by its bearing? That this is no extravagant Suppoſition, we have an Example in *Egypt* above-mentioned, where the *Nile* lays all flat before it, and ſo uniformly covers all with Mud, that there is no diſtinction. In ſuch a Caſe your Bearing muſt certainly help you out, there is no other way.

But I anſwer ſecondly, To ſay that the Surveyor might have determin'd the Point *Q* by admeaſurement from *G* and *H*, or any other adjoining noted Points, as from *F*, *K*, *I*, &c. 'tis very true; But then 'tis againſt our Suppoſition. I am upon ſhewing an Error that ariſes from judging of the Line *P Q by Magnetick bearing*, and to tell me that this might be avoided by another way, is to ſay nothing. I my ſelf ſhew how it may be avoided by allowing for the Variation; but ſtill it is an Error, till it be avoided.

But thirdly, If *B*'s Surveyor do not allow for the Variation of the Needle, he will never exactly determine even the Points *G*, *F*, *H*, *K*, &c. or any other Points in the Plat; but inſtead thereof will fall on the Points *g*, *h*, *f*, *k*.

From what has been laid down, we may ſee the abſolute neceſſity of allowing for the Variation of the Magnet, in comparing old Surveys with new ones; for want of which great Diſputes may ariſe between neighbouring Proprietors of Lands: and it were to be wiſh'd that our Honourable and Learned Judges would take this Matter into their Conſideration whenever any Buſineſs of this kind comes before them. Hitherto an abſolute Acquieſcence in the Down Survey, without any of the fore-mention'd Allowance, has been agreed upon as a ſtanding Rule in our Courts of Judicature in *Ireland*; but that many Men may be injured thereby, I ſuppoſe is manifeſt from what foregoes.

I have only this to add, That leaſt I be thought herein to ſtrike at the Truth or Exactneſs of the Down Survey, 'tis not at all the intention of this Paper, but rather to confirm it, by ſhewing which way Men ought to Examine it truly, and not by the common ways uſed by them, which rather confound it, and all that claim under it.

See the Table Fig. 6.

Although this Paper was chiefly deſigned for the ending of Conteſts in the Kingdom of *Ireland* about the intereſts of ſome of thoſe whoſe Lands are Neighbouring, and have been Surveyed by Magnetick Inſtruments, yet conſidering its univerſal Uſe, it was thought it would be very grateful to the Curious to publiſh it here.

Theorem Fig. 5. Transact. No. 230.

$$\overline{az+bz^2+cz^3+dz^4+ez^5+fz^6+gz^7+hz^8+iz^9\ \&c}\Big|^{m} =$$

$$= a^m z^m + \frac{m}{1}a^{m-1}bz^{m+1}$$

$$+ \frac{m}{1}\times\frac{m-1}{2}a^{m-2}b^2z^{m+2}$$
$$+ \frac{m}{1}a^{m-1}c$$

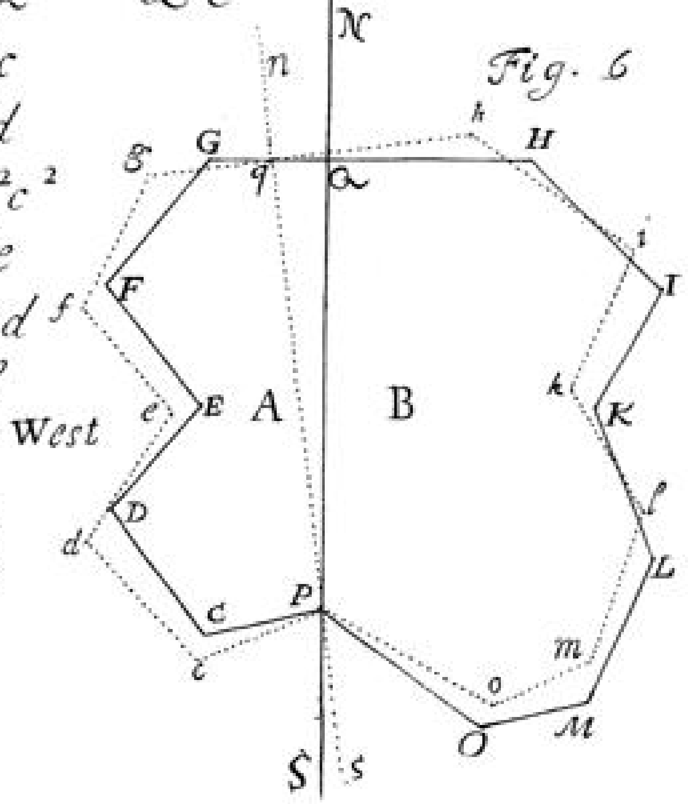
$$+ \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}a^{m-3}b^3z^{m+3}$$
$$+ \frac{m}{1}\times\frac{m-1}{1}a^{m-2}bc$$
$$+ \frac{m}{1}a^{m-1}d$$

$$+ \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}a^{m-4}b^4z^{m+4}$$
$$+ \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{1}a^{m-3}b^2c$$
$$+ \frac{m}{1}\times\frac{m-1}{1}a^{m-2}bd$$
$$+ \frac{m}{1}\times\frac{m-1}{2}a^{m-2}c^2$$
$$+ \frac{m}{1}a^{m-1}e$$

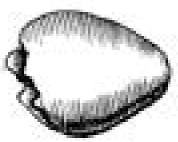
$$+ \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}\times\frac{m-4}{5}a^{m-5}b^5z^{m+5}$$
$$+ \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{1}a^{m-4}b^3c$$
$$+ \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{1}a^{m-3}b^2d$$
$$+ \frac{m}{1}\times\frac{m-1}{1}\times\frac{m-2}{2}a^{m-3}bc^2$$
$$+ \frac{m}{1}\times\frac{m-1}{1}a^{m-2}be$$
$$+ \frac{m}{1}\times\frac{m-1}{1}a^{m-2}cd$$
$$+ \frac{m}{1}a^{m-1}f$$

$$+ \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}\times\frac{m-4}{5}\times\frac{m-5}{6}a^{m-6}b^6z^{m+6}\ \&c$$
$$+ \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}\times\frac{m-4}{1}a^{m-5}b^4c$$
$$+ \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{1}a^{m-4}b^3d$$
$$+ \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{1}\times\frac{m-3}{2}a^{m-4}b^2c^2$$
$$+ \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{1}a^{m-3}b^2e$$
$$+ \frac{m}{1}\times\frac{m-1}{1}\times\frac{m-2}{1}a^{m-3}bcd$$
$$+ \frac{m}{1}\times\frac{m-1}{1}a^{m-2}bf$$
$$+ \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}a^{m-3}c^3$$
$$+ \frac{m}{1}\times\frac{m-1}{1}a^{m-2}ce$$
$$+ \frac{m}{1}\times\frac{m-1}{2}a^{m-2}d^2$$
$$+ \frac{m}{1}a^{m-1}g$$

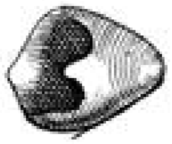
Fig. 6



4



3



2

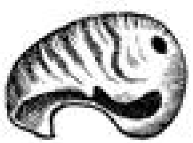


Fig. 1